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Dark energy and decompactification in string gas cosmology

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ABSTRACT: We study the stability of extra dimensions in string gas cosmology at late times. Vacuum energy and, interestingly, baryons lead to decompactification after they become dynamically important. The string gas can stabilise the effect of baryons, but not that of vacuum energy. However, we find that the interplay of baryons and strings can lead to acceleration in the visible dimensions, without the need for vacuum energy.

KEYWORDS: Cosmology of Theories beyond the SM.

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1. Introduction

String gas cosmology (SGC) [1–29] (see [29] for an introductory overview and a more comprehensive list of references) is a cosmological scenario motivated by string theory. In SGC, unlike in most applications of string theory, all spatial dimensions are treated on an equal footing: they are all compactified and start out small, and filled with a hot gas of branes of all allowed dimensionalities. Also in contrast to most higher-dimensional proposals, SGC aims to explain not only why some dimensions are hidden, but also why the number of visible dimensions is three (see [30,31] for other proposals along the same lines).

In the simplest versions of SGC there are nine spatial dimensions compactified on tori, all with initial sizes near the self-dual radius $\sqrt{\alpha'} \equiv l_s$. The branes can wind around the tori. The energy of the winding modes increases with expansion due to the tension of the branes, and this resists expansion. As the universe expands and cools down, winding and anti-winding modes annihilate, allowing further expansion. A simple counting argument suggests that p -branes and their anti-branes cannot find each other to annihilate in more than $2p + 1$ spatial dimensions, so at most $2p + 1$ dimensions can become large. For $p = 1$, corresponding to strings, this is three spatial dimensions. (Some quantitative studies of brane gases have cast doubt on this qualitative argument, see [4,9,10,13,19,20] for different analyses.)

It has been shown [15,23] that strings winding around the extra dimensions can stabilise them at the self-dual radius in the radiation dominated early universe while

the visible dimensions expand, even when the dilaton is stabilised (see also [1, 12, 18, 21, 24, 26, 28]). However, it was noted that the extra dimensions decompactify¹ if the universe is dominated by four-dimensional dark matter (a problem discussed early on in [1, 32]). It was concluded that ordinary dark matter has to be replaced by extra-dimensional dark matter (strings winding around the extra dimensions) to obtain viable late-time cosmology.

We take a closer look at late-time cosmology and decompactification. We note that even if dark matter is extra-dimensional, ordinary baryons will destabilise the extra dimensions after they become dynamically important. However, we find that the strings which stabilise the extra dimensions during the radiation dominated era can counter the destabilising push of four-dimensional pressureless matter, whether it is baryons or dark matter. The competition between the push of dust and the pull of strings cannot make the extra dimensions static, but it can lead to oscillations around the self-dual radius with decreasing amplitude, so that the extra dimensions can be regarded as effectively stabilised. Like dust, vacuum energy will lead to decompactification, but the strings cannot help there. However, we discover a possibility for obtaining acceleration without any extra dark energy component: the oscillations induced by dust and strings can involve transitions between deceleration and acceleration, even when the energy density of the universe is dominated by matter.

In section 2 we present the calculation of destabilisation by baryons and stabilisation by strings. We then show how the vacuum energy destabilises the extra dimensions and how the competition between four-dimensional matter and strings can lead to acceleration. In section 3 we discuss the relation to observations and summarise our results.

2. Dark energy and decompactification

2.1 Set-up

The metric and the equation of motion. We will consider a ten-dimensional spacetime, with all nine spatial directions compactified on tori². Six of the dimensions remain small, while three are large. We take the metric to be the simplest generalisation of the spatially flat Friedmann-Robertson-Walker universe, homogeneous and

¹We use the word 'decompactify' in the common, technically incorrect, sense of becoming macroscopically large. No change of topology is implied, and all spatial dimensions remain compact at all times.

²The important part about the topology is that it must have one-cycles for strings to wind around. For discussion of more complex compactifications, see [6, 7].

separately isotropic in the visible and the extra dimensions:

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^3 dx^i dx^i + b(t)^2 \sum_{j=1}^6 dx^j dx^j , \quad (2.1)$$

where $i = 1 \dots 3$ labels the visible dimensions and $j = 1 \dots 6$ labels the extra dimensions. Our convention is that for the small dimensions the value $b = 1$ corresponds to extra dimensions at the self-dual radius l_s . For the three large dimensions, we make the more convenient choice of $a = 1$ corresponding to the Big Bang Nucleosynthesis (BBN) era, specifically, to the time when $\rho_m/\rho_\gamma = 10^{-6}$ (and $\rho_\gamma \sim (\text{MeV})^4$).

We are interested in late-time behaviour, so we assume that the dilaton has been stabilised in a way that leaves the equation of motion of the metric unconstrained, so that it reduces to the Einstein equation (see [18, 21, 24, 26, 28])

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} , \quad (2.2)$$

where $G_{\mu\nu}$ is the Einstein tensor, κ^2 is the 10-dimensional gravitational coupling and $T_{\mu\nu}$ is the energy-momentum tensor (we lump the cosmological constant together with vacuum energy as part of the energy-momentum tensor).

Given the symmetries of the metric (2.1), the energy-momentum tensor has the form

$$T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t), P(t), P(t), P(t), P(t), P(t), P(t)) . \quad (2.3)$$

With (2.1) and (2.3), the Einstein equation (2.2) reads

$$\kappa^2 \rho = 3H_a^2 + 18H_a H_b + 15H_b^2 \quad (2.4)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\kappa^2(\rho + 3p) - \frac{3}{8}\kappa^2(\rho - 3p + 2P) + 6H_a H_b + 10H_b^2 \quad (2.5)$$

$$\kappa^2(\rho - 3p + 2P) = 8\frac{\ddot{b}}{b} + 24H_a H_b + 40H_b^2 , \quad (2.6)$$

where $H_a \equiv \dot{a}/a$ is the expansion rate of the three visible dimensions and $H_b \equiv \dot{b}/b$ is the expansion rate of the six extra dimensions.

When the extra dimensions are static, $H_b = 0$, we recover the usual FRW equations in the visible directions. In order to have $H_b = 0$, the driving term of b must vanish, $\rho - 3p + 2P = 0$.

The matter content. We will consider six kinds of matter. Ordinary four-dimensional radiation (γ), ordinary four-dimensional matter, also called dust (d), (consisting of baryons (b) and cold dark matter (cdm)) and vacuum energy (Λ) contribute to the energy-momentum tensor (2.2) with

$$\rho_\gamma = \rho_{\gamma, \text{in}} a^{-4} b^{-6} , \quad p_\gamma = \frac{1}{3} \rho_\gamma , \quad P_\gamma = 0 \quad (2.7)$$

$$\rho_b = \rho_{b,in} a^{-3} b^{-6}, \quad p_b = 0, \quad P_b = 0 \quad (2.8)$$

$$\rho_{cdm} = \rho_{cdm,in} a^{-3} b^{-6}, \quad p_{cdm} = 0, \quad P_{cdm} = 0 \quad (2.9)$$

$$\rho_\Lambda = -p_\Lambda = -P_\Lambda, \quad (2.10)$$

extra-dimensional dark matter (*edm*) obeys

$$\rho_{edm} = \rho_{edm,in} a^{-3} b^{-3}, \quad p_{edm} = 0, \quad P_{edm} = -\frac{1}{2}\rho_{edm}, \quad (2.11)$$

and finally, the string gas (*s*) has

$$\rho_s = M^{-1} \rho_{s,in} a^{-3} b^{-6} \sqrt{M^2 a^{-2} + b^{-2} + b^2 - 2} \quad (2.12)$$

$$p_s = \frac{1}{3} \frac{M^2 a^{-2}}{M^2 a^{-2} + b^{-2} + b^2 - 2} \rho_s \quad (2.13)$$

$$P_s = \frac{1}{6} \frac{b^{-2} - b^2}{M^2 a^{-2} + b^{-2} + b^2 - 2} \rho_s, \quad (2.14)$$

where the subscript *in* refers to the initial values, and M is the average initial energy of a string (due to momentum in the visible directions) in units of the string length l_s . (See [15, 23] regarding the extra-dimensional dark matter, and the appendix for the derivation of the string gas energy-momentum tensor.)

Baryons and dark matter, both *cdm* and *edm*, are always pressureless in the three large dimensions and will be collectively called matter (*m*), $\rho_m \equiv \rho_b + \rho_{cdm} + \rho_{edm}$. Four-dimensional matter, composed of baryons and cold dark matter, will be collectively called dust, $\rho_d \equiv \rho_b + \rho_{cdm}$. We introduce the dust fraction $f_d \equiv \rho_d / \rho_m$ to measure how much of the matter is four-dimensional. If all dark matter is extra-dimensional, $f_d = f_b \equiv \rho_b / \rho_m$, and if all dark matter is four-dimensional, $f_d = 1$. In principle, we could also have a mixture of *cdm* and *edm*, $1 > f_d > f_b$.

2.2 Dust and strings

Destabilising dust. We are interested in how the size of the extra dimensions behaves in the late-time universe, starting from the radiation dominated era, evolving to being dominated by dust and finally by vacuum energy.

When the universe is radiation dominated, the stability condition $\rho - 3p + 2P = 0$ is satisfied, and the extra dimensions stay at the self-dual radius, $b = 1$. When dust becomes important, the driving term $\rho - 3p + 2P$ becomes positive and b grows. From the four-dimensional point of view, the expansion of the extra dimensions looks like a time-dependent Newton's constant, with $8\pi G_N = \kappa^2 l_s^6 b^{-6}$. The change of Newton's constant from the initial value $G_{N,in}$ during BBN to the value today G_{N0} is constrained to be $G_{N0}/G_{N,in} = 0.98^{+0.25}_{-0.18}$ (2σ limit using ^4He and D abundance, and assuming negligible neutrino chemical potential) [33], which translates into $b_0 = 1.00^{+0.04}_{-0.03}$. This bound applies only to the final value, and does not limit b from being large between BBN and today. Allowing for a non-negligible

neutrino chemical potential, a combined analysis of BBN and the cosmological microwave background (CMB) leads to $1.38 > G_{N0}/G_{N,in} > 0.60$, which translates into $1.09 > b_0 > 0.95$ (2σ limit using older values for ${}^4\text{He}$ and D abundance) [34, 35]. Using information from the CMB makes the limit more model-dependent: in particular, the radiation degrees of freedom are assumed to be the same during BBN and at last scattering (which is in fact not true in the string gas model). Limits from CMB, large-scale structure and globular clusters [36] constrain the value of b between last scattering and today, but are strongly model-dependent. Even with the limits from BBN, we should take into account that the string gas provides additional radiation degrees of freedom, which can either tighten or relax the bounds (depending on how much the string gas contributes to the energy density during BBN and whether b is larger or smaller than unity today).

One caveat concerning the above limits is that the Newton's constant measured on Earth is not necessarily the cosmologically relevant quantity. This is both because the size of the extra dimensions may behave differently in regions where the visible dimensions are expanding and in regions which have broken away from the general expansion [37], and because the b -dependence in the Einstein equation does not factorise for all forms of matter. The latter point simply expresses the fact that the division between energy density and Newton's constant is one of convention³ (see [38] for discussion).

In [15, 23] it was concluded that because four-dimensional dark matter destabilises the extra dimensions, dark matter in SGC has to be extra-dimensional, i.e. if the pressure in the visible directions is zero, the pressure in the extra directions has to be negative to keep b static. Strings with winding around (but no momentum in) the extra dimensions were suggested as a dark matter candidate satisfying $p_{edm} = 0$, $P_{edm} = -\frac{1}{2}\rho_{edm}$, as listed in (2.11), so that $\rho_{edm} - 3p_{edm} + 2P_{edm} = 0$.

However, even if all dark matter was extra-dimensional, there would still be the four-dimensional baryons (as well as vacuum energy, which we discuss in section 2.3), with $p_b = P_b = 0$, to destabilise the extra dimensions. Note that while the extra-dimensional dark matter does not destabilise the extra dimensions, neither does it help to stabilise them. Figure 1 shows the evolution of a model with baryons and extra-dimensional dark matter (no vacuum energy). In this case the dust fraction is just the baryon fraction, which we take to initially be $f_d = f_b = 0.17$ (since $\rho_b/\rho_{edm} \propto b^{-3}$, the baryon fraction evolves with time). The extra dimensions start growing logarithmically after baryons become dynamically important. The size of the extra dimensions today (taken to be at 13.7 billion years, marked with the vertical line) is $b_0 = 1.3$, far in excess of the limits quoted above. However, the growth of the

³For example, $\rho_{edm} \propto b^{-3}$ in (2.11), so we could say that the effective four-dimensional gravitational coupling of the extra-dimensional matter goes like b^{-3} , in contrast to the b^{-6} behaviour of ordinary matter. Alternatively, we could say that the gravitational coupling is the standard one, but the energy density has an additional factor of b^3 .

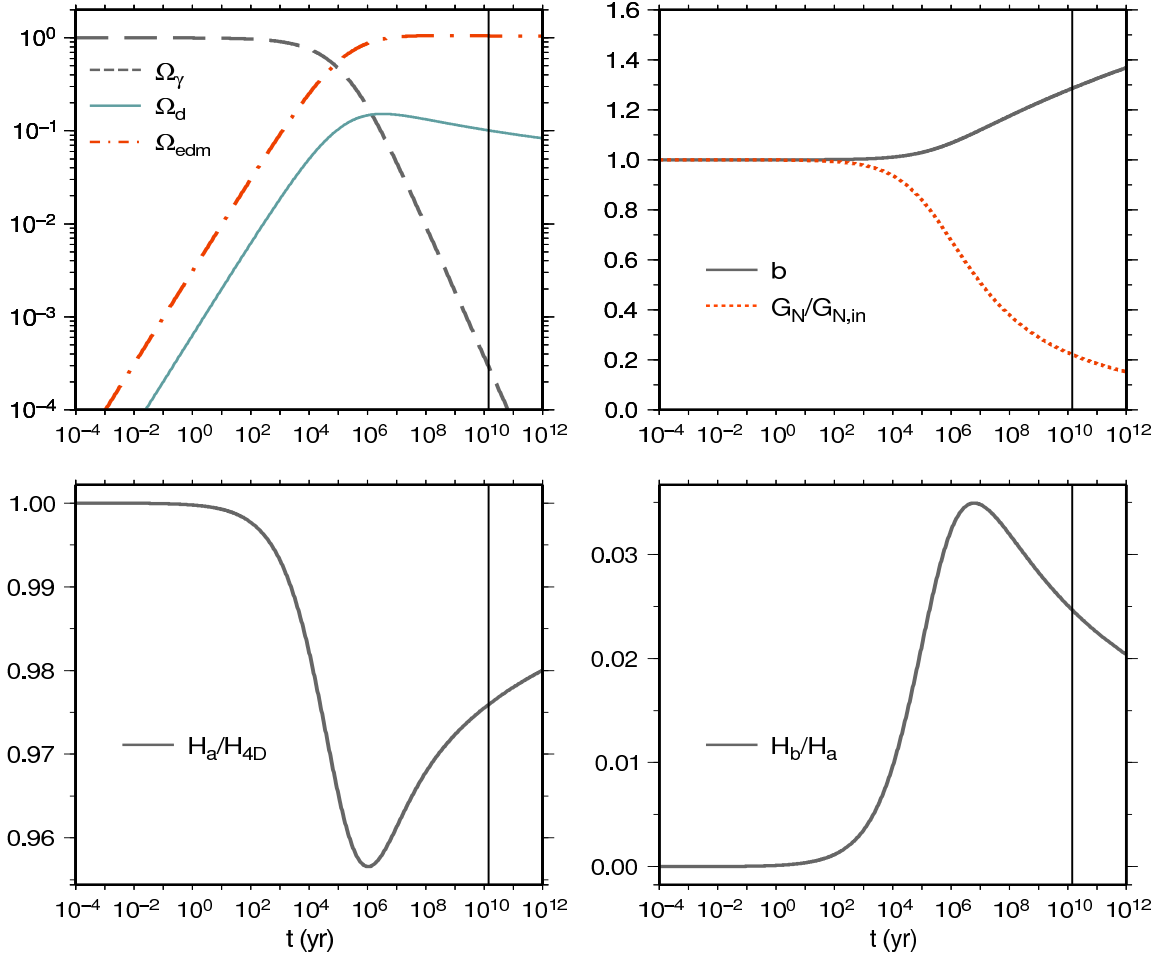


Figure 1: Density parameters $\Omega_i \equiv \kappa^2 \rho_i / (3H_a^2)$ (top left), the size of the extra dimensions and Newton's constant (top right), expansion rate of the large dimensions (H_{4D} is the Hubble parameter in the usual four-dimensional case) (bottom left) and the expansion rate of the extra dimensions (bottom right). All dark matter is extra-dimensional ($f_d = f_b = 0.17$) and there is no vacuum energy. The vertical line marks the present time $t_0 = 13.7$ Gyr.

extra dimensions due to baryons can be checked by the same gas of strings which stabilises the extra dimensions in the early universe.

Stabilising strings. The mechanism for driving the extra dimensions to the self-dual radius relies on a gas of strings having momentum and winding around the extra dimensions. The contribution of the string gas to the driving term of b is, from (2.12)–(2.14),

$$\rho_s - 3p_s + 2P_s = \frac{2}{3} \frac{2b^{-2} + b^2 - 3}{M^2 a^{-2} + b^{-2} + b^2 - 2} \rho_s. \quad (2.15)$$

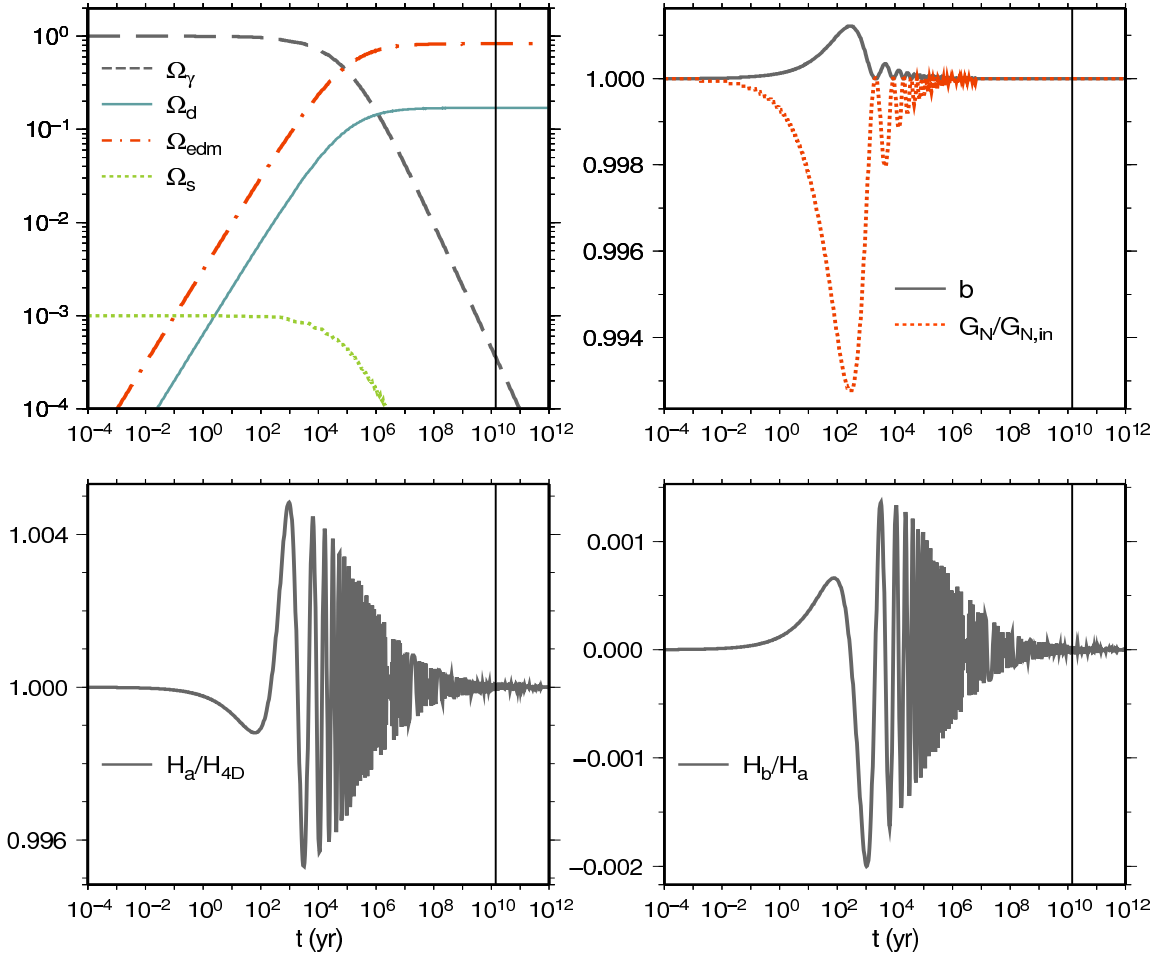


Figure 2: The model of Figure 1, with $f_d = 0.17$, now with stabilizing strings added. The initial density of the strings is $\Omega_{s,in} = 10^{-3}$ and $M = 588$.

During the radiation dominated era the extra dimensions remain static at $b = 1$, and the stabilising strings behave like four-dimensional radiation⁴, as (2.12)-(2.14) show, so their contribution to the driving term is zero. As the contribution of baryons to the driving term becomes important and pushes b up, the string contribution (2.15) becomes negative and tries to drive b downward (should b dip below 1, the string contribution will change sign and push b back up).

Two observations may be made about the stabilisation mechanism. First, $b = \sqrt{2}$ is a point of no return. If b grows beyond $\sqrt{2}$, the string contribution to the driving term becomes positive, so there is nothing to stop further expansion and the extra dimensions will decompactify. Second, there is no static solution for b . If the strings

⁴Their initial density is thus constrained by the limit on additional radiation degrees of freedom during BBN: $\Omega_{s,in} < 0.20$ with zero neutrino chemical potential [33], $\Omega_{s,in} < 0.40$ with a non-zero neutrino chemical potential [34] (both 2σ limits).

drive the extra dimensions back to the self-dual radius, baryons will destabilise them again. The extra dimensions either grow indefinitely or oscillate around the self-dual radius, but they cannot remain fixed at the self-dual radius.

Figure 2 shows the same model as before, with extra-dimensional dark matter and baryons (and no vacuum energy), but now with stabilising strings added. As expected, b oscillates near the self-dual radius, with a decreasing amplitude. Note that the strings can effectively stabilise the extra dimensions against the destabilising effect of baryons even when the contribution of strings to the total energy density is negligible (of the order 10^{-3} or less) throughout.

With regard to the dynamics of the extra dimensions, there is no difference between four-dimensional baryons and four-dimensional dark matter. So, the string gas can cure the destabilising effects of cold dark matter as well. Figure 3 shows the same model as before, but with extra-dimensional dark matter replaced with cold dark matter. Though the destabilisation is stronger than in the case with extra-dimensional dark matter, qualitatively the behaviour is the same as before.

Requiring that the strings turn the driving term of b negative before the point of no return at $b = \sqrt{2}$ leads to the limit

$$\frac{M^{-1}\rho_{s,in}}{\rho_{m,in}} > \frac{3}{2}f_d . \quad (2.16)$$

This limit is a necessary condition for stabilisation: unless (2.16) satisfied, the string gas contribution is too weak to overcome the baryons, and the extra dimensions will decompactify. However, it is not a sufficient condition, because if the driving term becomes negative too close to the point of no return, there isn't enough time to turn b around.

It may seem paradoxical in (2.16) that the strength of the stabilisation increases with decreasing initial string energy M . The reason is that, for a constant energy density, smaller energy means larger number density. When b is displaced from the self-dual radius, the energy that the extra-dimensional momentum and winding modes of a single string contribute is a fraction of the string scale l_s^{-1} , and does not depend on M . So, the stabilising contribution is proportional to the number density of strings and independent of their initial energy density.

To summarise, we find that SGC with cold dark matter can produce a matter-dominated era in agreement with observations, in contrast to the conclusion of [15, 23]. The same strings which stabilise the extra dimensions in the early universe can effectively stabilise them during the matter dominated era. Extra-dimensional dark matter then seems like an unnecessary complication. Having observed that dust poses no problem for late-time cosmology, we now take a look at vacuum energy, and discuss the relation between accelerated expansion of the visible dimensions and destabilisation of the extra dimensions.

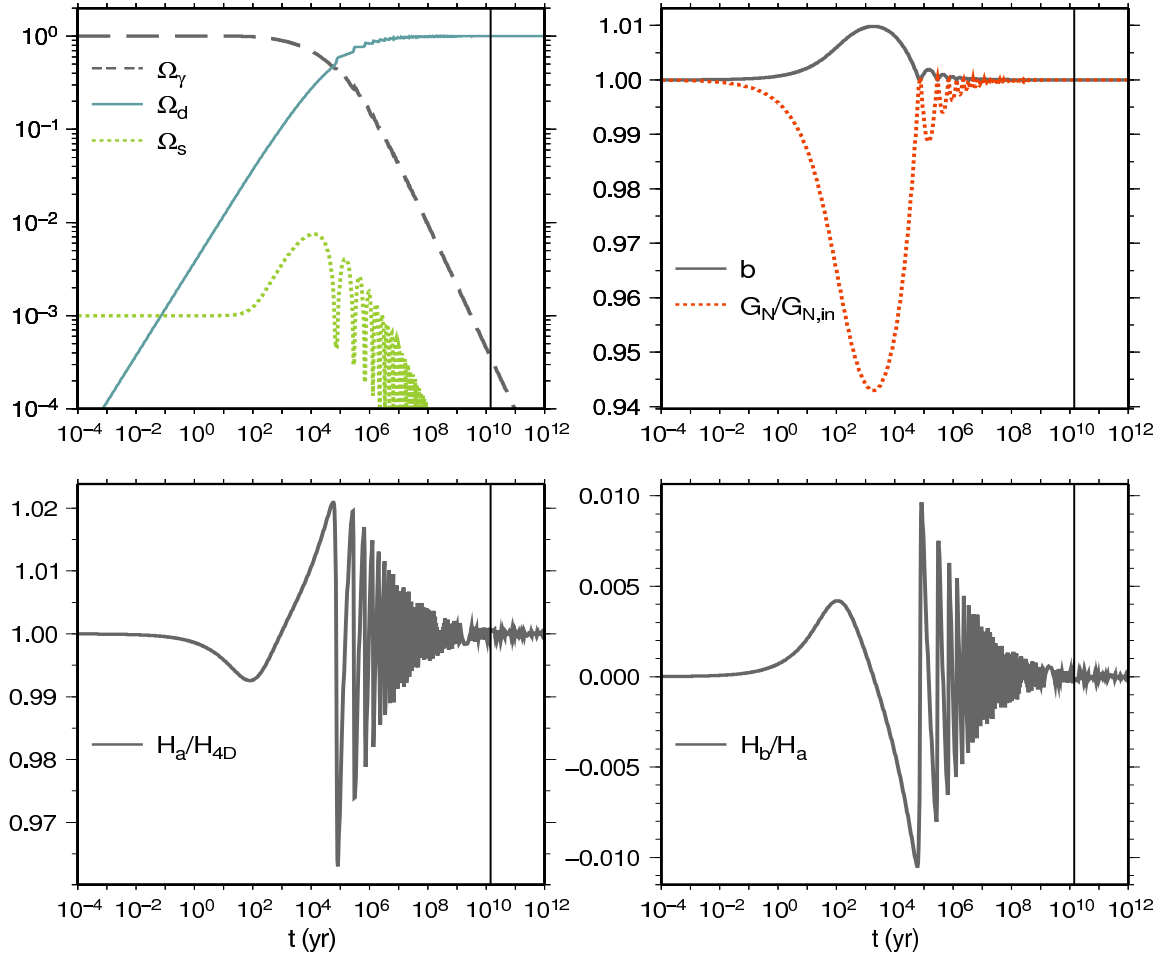


Figure 3: The same model as in Figure 2, but now with cold dark matter instead of extra-dimensional dark matter, $f_d = 1$.

2.3 Destabilisation and acceleration

Vacuum energy. Vacuum energy obeys the equation of state $p_\Lambda = P_\Lambda = -\rho_\Lambda$, so its contribution to the driving term is positive and will decompactify the extra dimensions. Figure 4 shows the evolution of a model with dust ($f_d = 1$) and vacuum energy in the 'concordance' proportions, and the same string gas as in Figure 1. (We take the 'concordance model' values $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ at 13.7 billion years [39] and extrapolate back to the BBN era when we give our initial conditions.)

The initial small oscillations of H_a and H_b are due to the interplay of dust and the string gas, and are the same as seen in Figure 3. When vacuum energy becomes important, b starts growing exponentially and the visible and extra dimensions rapidly isotropise as they undergo exponential inflation, in agreement with the no-hair conjecture [40]. As in the case of baryons, the growth of b can in principle be used to rule the model out. For the concordance values, we have $b_0 = 1.06$ today, inside the 2σ

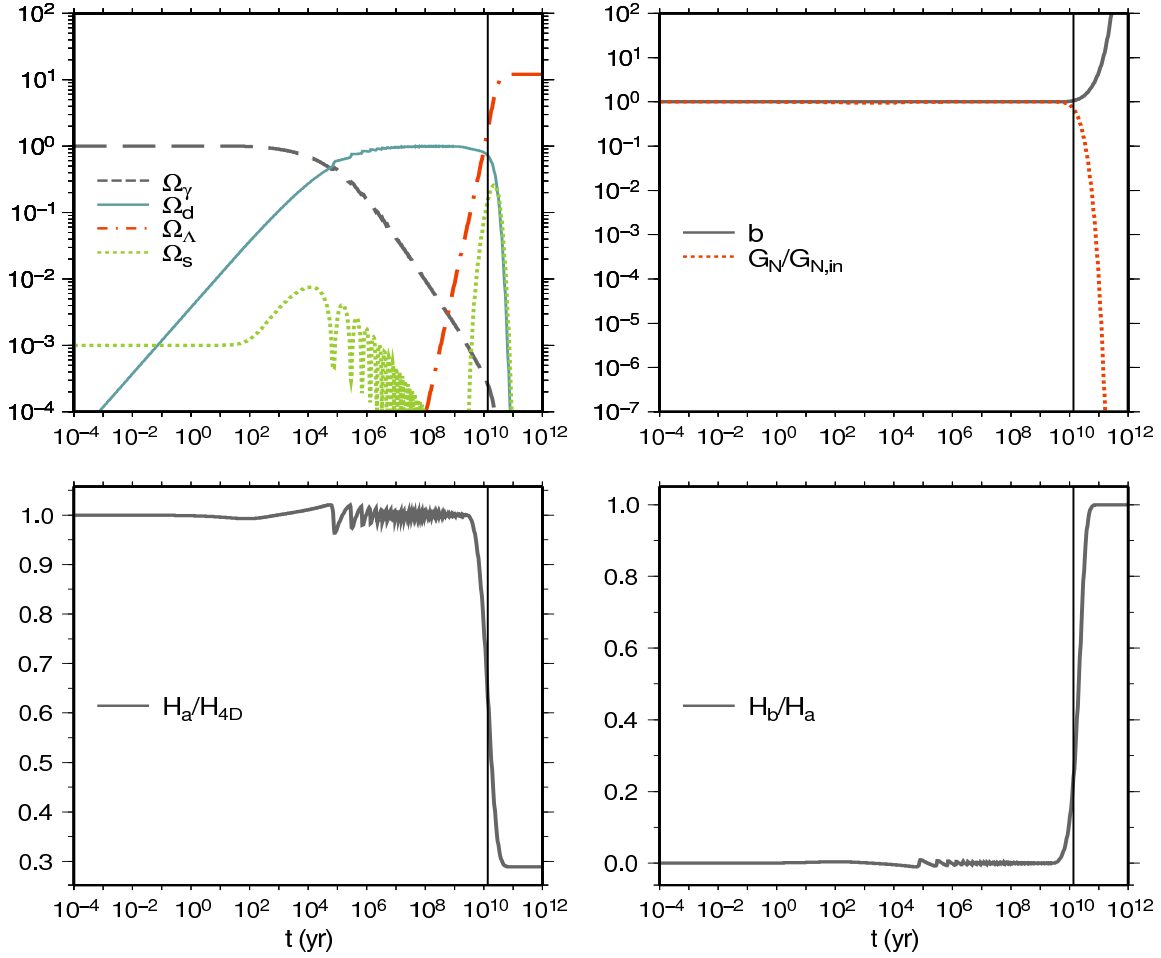


Figure 4: A model with ‘concordance’ proportions of dust ($f_d = 1$) and vacuum energy, and the same stabilising strings as in Figures 2 and 3.

limit from BBN (when allowing for a neutrino chemical potential). Tighter bounds on the electron neutrino chemical potential, a larger age of the universe (or perhaps simply redoing the neutrino chemical potential analysis of [34] with the updated abundances in [33]) could rule the model out.

In contrast to the gentle push from baryons which led to logarithmic growth of b , the dramatic destabilisation due to vacuum energy cannot be prevented by the string gas. In fact, destabilisation of the extra dimensions is a general feature of acceleration of the visible dimensions. Assuming that the extra dimensions are static implies $\rho - 3p + 2P = 0$. Acceleration in the visible dimensions then requires $\rho + 3p < 0$, which leads to $\rho + P < 0$. So, acceleration implies that the extra dimensions are dynamical or the null energy condition is violated or both. This is in agreement with the observation in [23] that a period of scalar-field driven inflation would destabilise the extra dimensions. (For discussion of inflation in SGC, see [11, 17, 22, 25].)

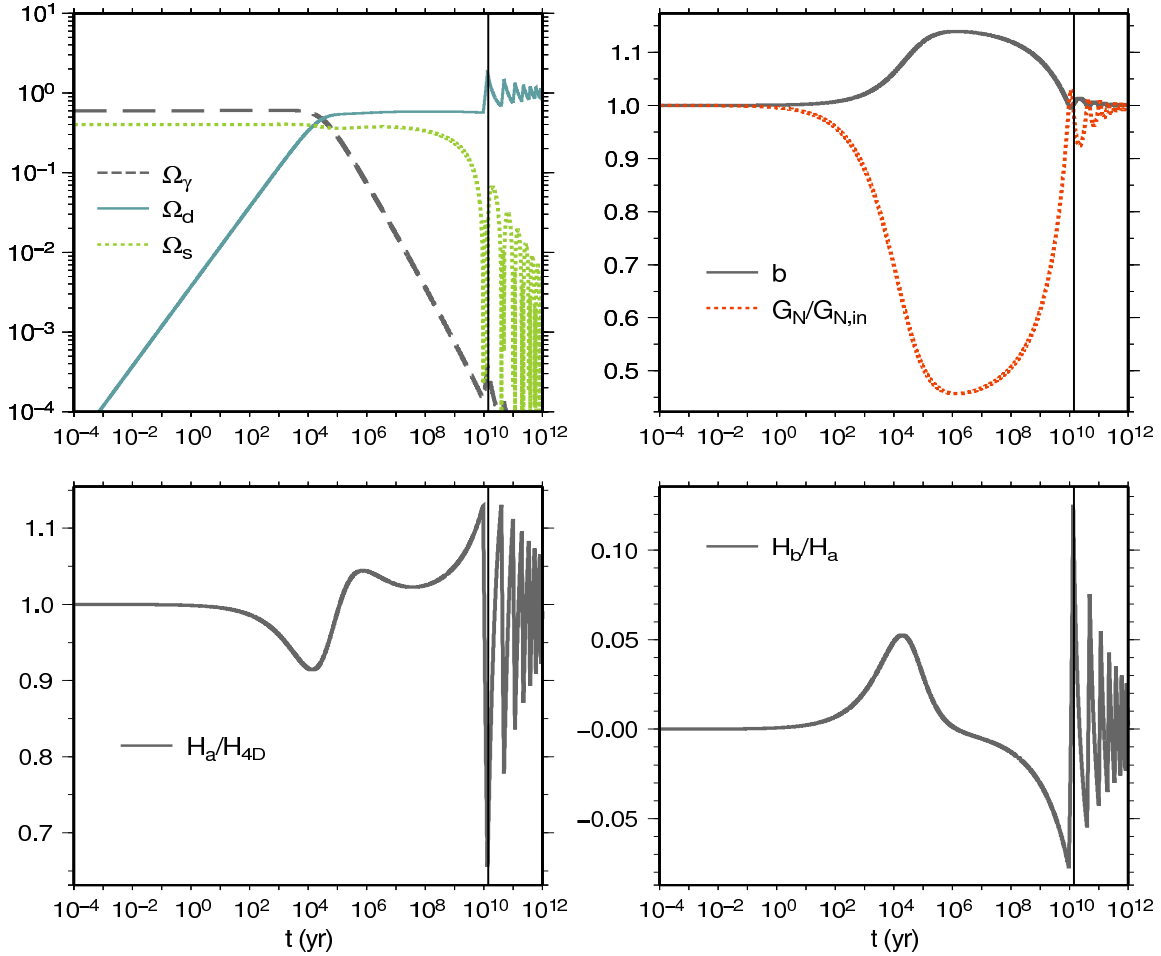


Figure 5: An accelerating model with dust ($f_d = 1$) and strings ($\Omega_{s,in} = 0.40$, $M = 158\,730$).

It might seem that SGC is under stringent constraints due to the change in b implied by acceleration⁵, and that it would be ruled out by a small tightening of the bounds – at least in the simplest setting where the extra dimensions are toroidal, isotropic, spatially flat and stabilised by the string gas (changing vacuum energy to a dark energy model with more parameters could also allow some breathing room). However, there is a surprising way out: though the stabilising strings cannot prevent decompactification in the case of vacuum energy, they can lead to acceleration without decompactification in the absence of vacuum energy (though this will turn out to involve violating the null energy condition).

Dust. Acceleration leads to destabilisation of the extra dimensions, so we can ask whether it is possible to obtain acceleration as a result of destabilising the extra

⁵Unless observations could be fitted without acceleration, see [41] for an interesting suggestion and also [42].

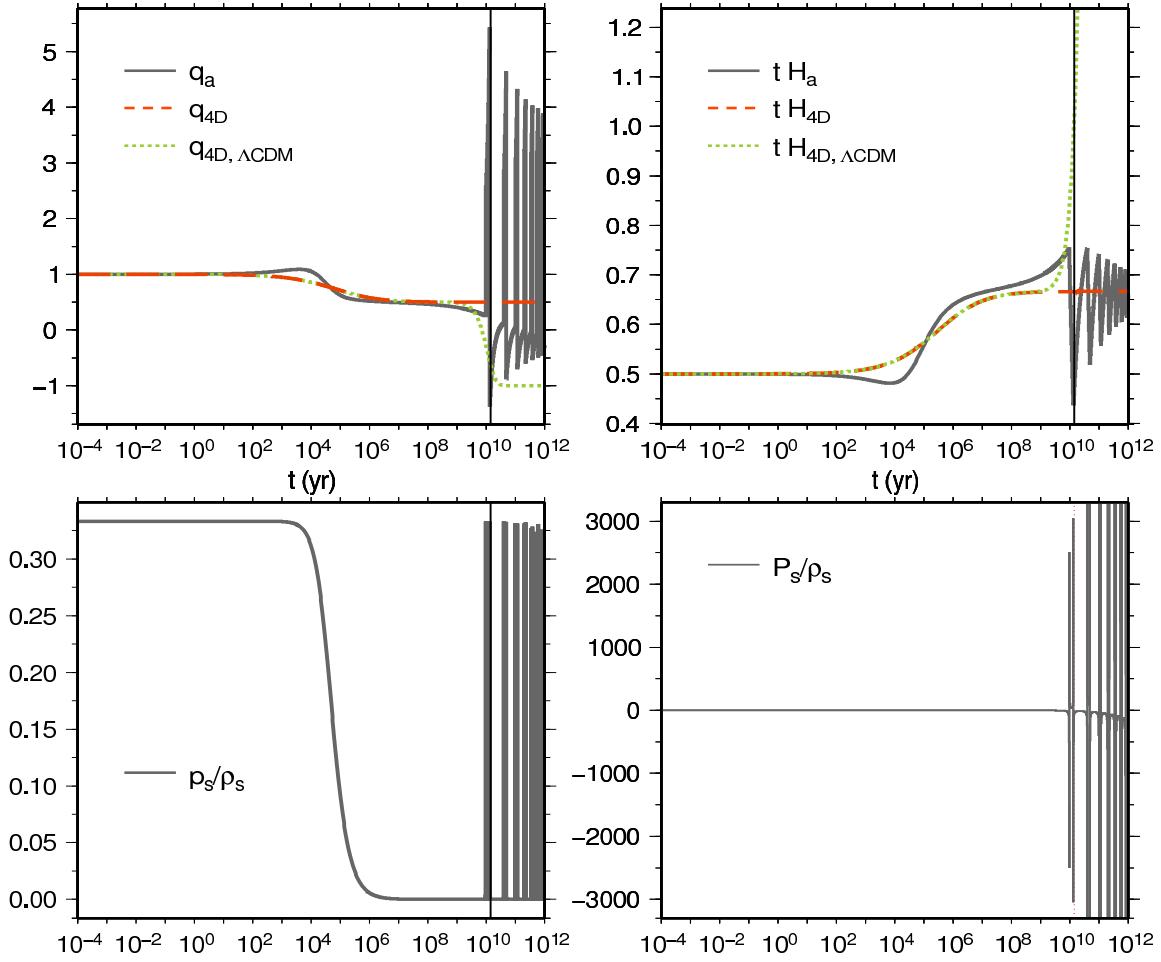


Figure 6: The deceleration parameter (top left), the expansion rate of the visible dimensions with regard to age (top right), pressure in the visible dimensions (bottom left), pressure in the extra dimensions (bottom right). The model is the same as in Figure 5.

dimensions, rather than vice versa. Let us assume that the vacuum energy is zero. As dust becomes dynamically important, it will push the extra dimensions to expand. The stabilising strings will pull the extra dimensions back and will eventually turn b around (if their number density is large enough). The acceleration equation (2.5) shows that a negative driving term for b contributes positively to the acceleration of a , suggesting that a collapsing extra dimension could lead to acceleration. There is also a negative contribution to the acceleration from the terms involving H_b (for $|H_b| < 3/5 H_a$), so it is not immediately clear what will happen.

Figure 5 shows the evolution of a model with baryons, cold dark matter ($f_d = 1$) and stabilising strings. As in Figures 2 and 3, the strings can have a major impact on the dynamics even when their contribution to the total energy density is negligible ($\rho_s/\rho < 10^{-2}$ today). First the extra dimensions open up and the expansion rate

of the visible dimensions slows down (relative to the 4D case). When the extra dimensions turn around, the expansion rate of the visible dimensions speeds up as b collapses, and reaches a maximum at the minimum value of H_b . As b dips below 1, the strings rapidly bounce it back, and b starts oscillating around the self-dual radius as in Figures 2 and 3. The amplitude of the oscillations decreases rapidly, and today we have $b_0 = 1.002$, within the observational limits (even when also taking the contribution of the string energy density into account). However, now the oscillations also involve going back and forth between acceleration and deceleration. The deceleration parameter $q = -\ddot{a}/a/H_a^2$ is shown in the top left panel of Figure 6.

Not only can the string gas stabilise the extra dimensions in the matter-dominated era, but it can also lead to late-time acceleration in the process. This seems particularly noteworthy given that we have added no new ingredients to SGC, simply taken account of the fact that baryons and cold dark matter are four-dimensional. However, the rapidly oscillating behaviour shown in Figures 5 and 6 is quite different from the smooth change from deceleration to acceleration in the Λ CDM model, and we will now discuss the comparison to observations.

3. Discussion

3.1 Phenomenology

In order to compare the model to observations, one has to solve the equations (2.4)–(2.6) with the matter content given by (2.7)–(2.14) numerically for each value of $\Omega_{s,in}$, M and f_d to obtain $a(t)$ to compare with observations (and $b(t)$ to check it is not excluded by observations). Comparison to SNIa data would be relatively straightforward, while comparison to CMB and large-scale structure would require extending the equations beyond the homogeneous level to perturbation theory [14, 16, 27]. We leave a thorough study of the $(\Omega_{s,in}, M, f_d)$ -parameter space and detailed comparison to observations for later work, and only make some mostly qualitative comments here.

Since the extra dimensions start opening up when matter becomes dynamically important, the acceleration naturally starts only after the matter-radiation equality. (Also, it is more difficult for oscillations to reach into the accelerating regime during the radiation-dominated era because q is larger.) This is reminiscent of tracker models [43], and the strings do go from behaving like radiation to behaving like matter as the universe goes from radiation domination to matter domination, if the growth of b is slow (as one can see from the plot of p_s/ρ_s in the bottom left panel of Figure 6, or by expanding (2.12) around $b = 1$). With regard to the coincidence problem, the model has the same shortcoming as tracker fields: the preferred time is shortly after the matter-radiation equality at $t_{eq} \sim 10^5$ years, but according to observations the acceleration starts much later, at around 10 billion years $\sim 10^5 t_{eq}$.

The acceleration starts as the collapse of the extra dimensions ends with b starting to oscillate. So, the later b turns around from expansion from collapse, the later the acceleration starts. The latest possible turnaround is achieved for models where b is at the limit of expanding forever instead of turning around. A large peak value of b also seems to help by making the end of the collapsing period violent enough that the resulting oscillations reach deep into the accelerating domain, as required by observations ($q_0 \leq -0.3$ [44]). For the accelerating model shown in Figure 5 with $M = 158\,730$, the extra dimensions would decompactify for the value $M = 160\,000$, and for smaller values the acceleration would start earlier (for example, for $M = 50\,000$ it starts between 10^5 and 10^6 years). So, while it is natural for the acceleration to be in the late era of cosmology, starting as late as $\sim 10^5 t_{eq}$ requires tuning.

Another possibility could be that the oscillations have indeed started early, and that we are not seeing the first stage of acceleration. However, in the stabilised models we have looked at, the quantity tH_a stays near the 3D matter-dominated value $2/3$ already after a few oscillations, as shown in the top right panel of Figure 6. In other words, the periods of acceleration and periods of extra deceleration cancel each other out. The value today $t_0 H_{a0}$ can be written as $1.3h \times t_0 / (13 \text{ billion years})$, where $H_{a0} = h \, 100 \text{ km/s/Mpc}$, which for the ‘concordance’ values $h = 0.71$, $t_0 = 13.7$ billion years gives 0.99. In Figure 6, the maximum value of tH_a is only 0.75 at the first oscillation, going down thereafter. Further, the minima of tH_a and q coincide, so getting a large enough tH_a simultaneously with a negative enough q to match observations is increasingly difficult for later oscillations. Even for the first transition to acceleration it looks desirable to shift the age of the universe to optimise the fit of q and tH_a ,⁶ though we emphasise that we have not done a comprehensive search of the parameter space to find the best-fitting model. Note that most parametrisations used to fit the SNIa data would not see rapid oscillations, in which case we should rather consider the average over oscillations (the scatter of the unbinned data points is large, so it is not clear how well one can detect small-frequency oscillations with the current data). If the oscillations are not too rapid to see individually in the data, then the sharp transition to acceleration, whether it is the first or a subsequent one, is a distinctive signature. A sharp transition is allowed, but not required, by the current data [45–48]. Note that constraints derived for a rapid transition (see e.g. [49]) often depend on the chosen parametrisation for the equation of state [47, 48].

The values of M we have used correspond to very high energies: the strings used in Figures 5 and 6 have an average momentum of $M \sim 10^5 l_s^{-1}$ at the BBN era. At first sight, this may seem unnatural, since for point particles in thermal equilibrium the number density of high-energy states goes down exponentially. However, for a gas of strings at high energies in a compact space, the existence of winding modes

⁶Though see [42] regarding the observational value of the Hubble parameter.

makes the energy concentrate in a small number of highly energetic strings [50–52], as discussed in the appendix. It is amusing that instead of having to input an unnaturally low energy scale for the acceleration to start late enough, as in most dark energy models⁷, we need an extremely high energy scale, which arises naturally in string thermodynamics in compact spaces.

Another parameter which seems large is the initial energy density of strings, which we have pushed to the limit allowed by BBN bounds (to make the effect of the strings clearer). From the limited studies of the parameter space we have done, it seems that the minimum value of q depends on $\Omega_{s,in}$ (unlike the strength of the stabilisation, which depends on the number density). For $f_d = 1$, at least $\Omega_{s,in} = 0.10$ still results in acceleration.

It is noteworthy that the deceleration parameter can dip below the de Sitter value -1 . Such rapid acceleration is usually associated with violation of the null energy condition, i.e. equations of state more negative than -1 . Since the departure from matter-dominated 4D behaviour is due to both the extra dimensions and the string gas, the apparent dark energy does not obey a simple equation of state. However, it is still true that the violation of the null energy condition by the strings is an essential ingredient of the acceleration (it allows the strings to have a major impact on the dynamics even when their energy density is negligible). The bottom right panel in Figure 6 shows P_s/ρ_s , which reaches values of over 100 and below -720 in the period until today. In [15, 23] it was argued that the dominant energy condition is not violated because the strings have a large momentum in the visible directions (compared to $\sqrt{b^{-2} + b^2 - 2}l_s$). The violation of the dominant and null energy conditions shown in Figure 6 is indeed related to the momentum in the visible directions becoming small compared to $\sqrt{b^{-2} + b^2 - 2}l_s$, as can be seen by expanding (2.14) around $b = 1$.

An interesting feature related to the violation of the null energy condition is that ordinary matter dominates the energy density even when the universe is accelerating, so we could in principle have $\Omega_{tot} \approx \Omega_m = 0.2 \dots 0.3$ today (this requires $H_b/H_a = -0.13 \dots -0.15$, and is not realised in the model shown in Figures 5 and 6). However, this does not imply spatial curvature, since the correspondence between spatial flatness and critical density is broken by the extra-dimensional terms in the Hubble law (2.4): $\Omega_m \equiv \kappa^2 \rho_m / (3H_a^2) \neq \rho_m / \rho$.

3.2 Conclusion

We have studied the stabilisation of the extra dimensions in late-time string gas cosmology (SGC). In addition to the destabilisation by four-dimensional dark matter noted in [15, 23], baryons will drive the extra dimensions to expand. The effect of both baryons and four-dimensional dark matter can be checked by the string gas

⁷See [53] for an interesting idea for avoiding the introduction of a low energy scale.

which has stabilised the extra dimensions in the early universe. However, vacuum energy, or any other dark energy candidate satisfying the null energy condition, will rapidly decompactify the extra dimensions.

We find that SGC has a built-in mechanism for producing late-time acceleration which doesn't decompactify the extra dimensions or require vacuum energy. The interplay between matter pushing the extra dimensions to expand and strings reining them in leads to oscillations around the self-dual radius. This can involve oscillations between deceleration and acceleration, depending on the number density and energy density of the string gas. The violation of the null energy condition by the strings makes it possible to have acceleration even when the energy density of the universe is dominated by ordinary matter, with $\Omega_{tot} \approx \Omega_m < 1$, without contradicting spatial flatness.

Showing that a matter-dominated period followed by accelerated expansion without decompactification is possible may be seen as a step towards developing SGC into a realistic model of the universe at all eras. However, it is not clear whether the late-time acceleration produced by this mechanism can be in agreement with observations, and we leave a detailed study of the parameter space of the model and comparison to observations for future work.

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A. Appendix

In this appendix we derive the energy-momentum tensor of the string gas, (2.12)-(2.14). Following [15, 23], we consider a gas of strings with winding and momentum modes in the extra dimensions, and only momentum modes in the visible dimensions (the winding modes in the visible dimensions are assumed to have annihilated). Given the metric (2.1), and assuming that all dimensions are compactified on tori, the energy of a string state is given by⁸

$$l_s^2 E^2 = a^{-2} \sum_{i=1}^3 N_i^2 + \sum_{j=1}^6 (b^{-1} N_j + b W_j)^2 + 4(N - 1) , \quad (\text{A.1})$$

⁸In the present context, this quantity is properly called the energy rather than the mass, since we are not looking at the states from the four-dimensional Kaluza-Klein point of view.

where $l_s \equiv \sqrt{\alpha'}$, the integers N_i are the momentum quantum numbers of the visible directions, N_j, W_j are the momentum and winding quantum numbers, respectively, of the extra directions, the non-negative integer N is the oscillator level, and the quantum numbers are subject to the level matching constraint $N + \sum_{j=1}^6 N_j W_j \geq 0$. The normalisation is such that $a = b = 1$ corresponds to dimensions at the self-dual radius. In contrast to [15,23], all momenta are quantised, since we take all dimensions to be compact (in the late universe where the visible dimensions are large and there are no modes winding around them, this doesn't make any difference). The validity of (A.1) requires that a and b change slowly compared to the string scale l_s (see the appendix of [15] for details).

In [15,23], only states with vanishing contribution to the energy from the oscillator modes and the extra-dimensional momentum and winding modes at the self-dual radius (“massless states”) were taken into account. (It was noted in [26] that in type II superstring theory such states are removed by the GSO projection, but they are present in heterotic string theory.) Those states which do not remain at zero energy perturbatively near the self-dual radius were then discarded. We follow slightly different reasoning. Some of the states with vanishing energy at the self-dual radius (for example, ones with $\alpha' E^2 = 3b^{-2} + b^2 - 4$) become tachyonic away from the self-dual radius. With the momenta in the visible dimensions quantised, some states even become tachyonic when the extra dimensions are stabilised at the self-dual radius and the visible dimensions expand (for example, ones with $\alpha' E^2 = 2a^{-2} + b^{-2} + b^2 - 4$). (Some other issues regarding the case when all momenta are quantised are discussed in [20].) We keep only the states for which the contribution to the energy from the oscillator modes and the extra-dimensional momentum and winding modes at the self-dual radius vanishes, and which are never tachyonic. (The requirement of not being tachyonic also removes the states which have zero energy at half-integer fractions and half-integer multiples of the self-dual radius, discussed in [15].) We are left with four sets of quantum numbers: (1) $N = 1, N_j = W_j = 0$ for all j , (2) $N = 1, N_j = -W_j = \pm 1$ for one j and zero for others, (3) $N = 0, N_j = W_j = \pm 1$ for one j and zero for others and (4) $N = 0, N_{j_1} = \pm 1, W_{j_2} = \pm 1$ for two values of j_1 and j_2 and zero for others, such that $\sum_{j=1}^6 N_j W_j = 0$. Excluding tachyonic states leaves almost the same set of states as requiring zero energy from extra-dimensional and oscillator modes perturbatively near the self-dual radius. (In [23], the subset of the last category of states where $j_1 \neq j_2$ for all entries were also discarded, since they do not have zero energy when the perturbations of the extra dimensions are anisotropic. However, with isotropic extra dimensions, there is no reason to discard them.) The energy density of a gas of strings in states with these quantum numbers is

$$\rho = \sum_{\text{states}} n_{\text{state}} E_{\text{state}}$$

$$\begin{aligned}
&= \sum_{N_1^{(1)}, N_2^{(1)}, N_3^{(1)}} n_{(1), N_1^{(1)} N_2^{(1)} N_3^{(1)}} \sqrt{\sum_{i=1}^3 N_i^{(1)2} a^{-1} l_s^{-1}} \\
&\quad + 12 \sum_{\{N_i^{(2)}\}} n_{(2), \{N_i^{(2)}\}} \sqrt{\sum_{i=1}^3 N_i^{(2)2} a^{-2} + b^{-2} + b^2 - 2 l_s^{-1}} \\
&\quad + 12 \sum_{\{N_i^{(3)}\}} n_{(3), \{N_i^{(3)}\}} \sqrt{\sum_{i=1}^3 N_i^{(3)2} a^{-2} + b^{-2} + b^2 - 2 l_s^{-1}} \\
&\quad + 300 \sum_{\{N_i^{(4)}\}} n_{(4), \{N_i^{(4)}\}} \sqrt{\sum_{i=1}^3 N_i^{(4)2} a^{-2} + 2b^{-2} + 2b^2 - 4 l_s^{-1}}, \quad (\text{A.2})
\end{aligned}$$

where n is the number density, $N_i^{(q)}$ is the momentum number in the direction $i = 1, 2, 3$ for a state which has oscillator and extra-dimensional momentum and winding quantum numbers identified by $q = 1, 2, 3, 4$ (corresponding to the four possibilities listed above), $n_{q, \{N_i^{(q)}\}}$ is the number density of that state and the coefficients 12 and 300 are the multiplicities of the states.

In thermal equilibrium, the number density $n_{q, \{N_i^{(q)}\}}$ is given by the occupation number of the state divided by the volume of the manifold, $l_s^9 a^3 b^6$. We ignore possible time-dependence of the occupation numbers. The first sum in (A.2) then behaves like four-dimensional radiation, with all terms proportional to a^{-1} . It brings nothing new to the analysis compared to ordinary radiation, so we neglect it. The other terms, which include contributions from both visible and extra dimensions, are more cumbersome. The sum does not reduce to a single term, but the various terms are qualitatively the same, so we replace the sum with a single representative term. The energy density and the pressures then read

$$\rho = \frac{1}{a^3 b^6} n_{s, in} l_s^{-1} \sqrt{M^2 a^{-2} + b^{-2} + b^2 - 2} \quad (\text{A.3})$$

$$p = \frac{1}{3} \frac{1}{a^3 b^6} n_{s, in} l_s^{-1} \frac{M^2 a^{-2}}{\sqrt{M^2 a^{-2} + b^{-2} + b^2 - 2}} \quad (\text{A.4})$$

$$P = \frac{1}{6} \frac{1}{a^3 b^6} n_{s, in} l_s^{-1} \frac{b^{-2} - b^2}{\sqrt{M^2 a^{-2} + b^{-2} + b^2 - 2}}, \quad (\text{A.5})$$

where M is the initial average energy of a string in units of l_s^{-1} and $n_{s, in}$ is the initial number density, both at the time when $a = b = 1$. Identifying $n_{s, in} = \rho_{s, in} / (M l_s^{-1})$, we have (2.12)-(2.14). The absence of winding modes around the visible dimensions allows us to rescale a and M so that $a = 1$ corresponds to any convenient era. We have chosen to set $a = 1$ at the BBN, when $\rho_m / \rho_\gamma = 10^{-6}$ (and $\rho_\gamma \sim (\text{MeV})^4$).

Instead of taking a single term, we could have averaged over the quantum numbers with the correct number density. The number density of strings in thermal

equilibrium in a toroidal compact space where three dimensions grow large has been calculated in [52] (see also [50, 51]). For high energies, the number density falls like $(\text{energy})^{-1}$, so the relative contribution of high-energy modes to the energy density is almost independent of their energy (this conclusion depends on all dimensions, including the large ones, being compact and admitting one-cycles). The energy is concentrated in a small number of highly energetic strings, which is the qualitative picture needed for strings to cause late-time acceleration. However, the results cannot be applied directly to the present case, since we have assumed that the modes winding around the large dimensions have annihilated, unlike in [52]. The number density also depends on the detailed behaviour of the early universe, particularly on how the strings are produced and how they thermalise. It may be that the temperature at which the strings are produced is too low for the distributions derived in [50–52] to be relevant. It is also possible that the strings have never been in thermal equilibrium, and that their number density is not determined by thermodynamical arguments. For discussion of thermodynamics in string gas cosmology, see [1, 2, 4, 10, 13, 19, 20].

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